4)
$F'(x) = f(x) > 0$

x	1	$+\infty$
$F'(x)$	+	
$F(x)$	Ln 2 ↗	$+\infty$

* $F(x) \geq x \operatorname{Ln} 2$ pour $x > 1$

$\lim\limits_{x \to +\infty} x.Ln(2) = +\infty$

D'où $\lim\limits_{x \to +\infty} F(x) = +\infty$

* déterminons $\lim\limits_{x \to +\infty} \dfrac{F(x)}{x}$

Pour $x > 1$

$x \leq t \leq x^2 \Rightarrow \operatorname{Ln} x \leq \operatorname{Ln} t \leq \operatorname{Ln} x^2$

$\Rightarrow \dfrac{1}{2\operatorname{Ln} x} \leq \dfrac{1}{\operatorname{Ln} t} \leq \dfrac{1}{\operatorname{Ln} x}$

$\rightarrow \int_x^{x^2} \dfrac{1}{2\operatorname{Ln} x}\, dt \leq F(x) \leq \int_x^{x^2} \dfrac{dt}{\operatorname{Ln} x}$

$\Rightarrow \dfrac{x^2 - x}{2\operatorname{Ln} x} \leq F(x) \leq \dfrac{x^2 - x}{\operatorname{Ln} x}$

$\Rightarrow F(x) \geq \dfrac{x^2 - x}{2\operatorname{Ln} x}$

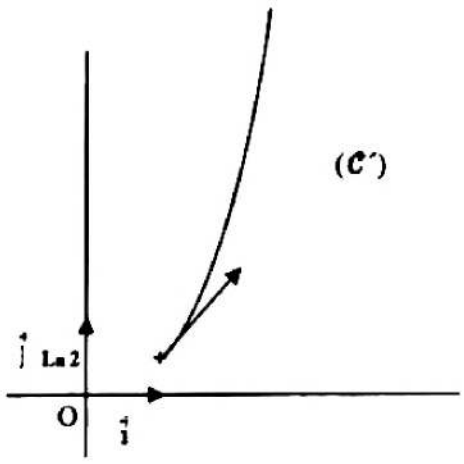
$\Rightarrow \dfrac{F(x)}{x} \geq \dfrac{x - 1}{2\operatorname{Ln} x}$

$\lim_{x \to +\infty} \dfrac{x - 1}{2\operatorname{Ln} x} =$

$\lim_{x \to +\infty} \dfrac{1}{2} f(x) = +\infty$

D'où $\lim_{x \to +\infty} \dfrac{\Gamma(x)}{x} = +\infty$

$(\mathcal{C}')$ admet une branche parabolique de direction $(O, \vec{j})$ au voisinage de $+\infty$



III) $\alpha > 1$

1) pour tout $x \in]1, +\infty[$

$F(x) = \int_x^{x^2} \dfrac{dt}{\operatorname{Ln} t}$

Soit $G(x) = \int_1^x f(t)\, dt + \operatorname{Ln} 2$

On a : $F'(x) = f(x)$

$G'(x) = f(x)$

D'où : $F(x) = G(x) + k$, $k \in IR$

$F(1) = G(1) + k \Rightarrow k = 0$

d'où

$F(x) = G(x)$; $\forall x \in]1, +\infty[$

$F(x) = \int_1^x f(t)\, dt + \operatorname{Ln} 2$

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